THE SLIDE RULE

BY

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Den Polytekniske Læreanstalt

DIRECTIONS FOR THE USE OF THE "SLIDE RULE"

venskabeligst fra dens forhenv. Elev

BY

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M. Danish I.C.E.

The slide rule is a graphical representation of the common logarithms (Briggs), two scales on top and one of double length at bottom. Hence going to the right from one top scale to another or to an imaginary prolongation of the bottom scale, we get the characteristic aug„ „mented by 1.* The moveable index, called the cursor, serves to mark any number on the fixed scale if it is wanted to keep it while moving the slide. Subdivisions are made by judgement.

A dash and dot line in any of the following figures indicates the place where the results of the calculations are read.

*All ought to be read with a slide rule at hand.

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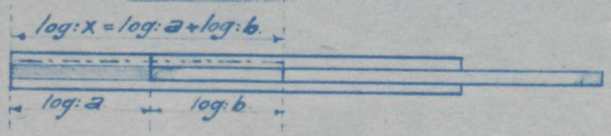
*The alteration of 1 in the characteristic is pointed out by Claudel and forms the basis of following rules for the charact.c made out by the author

MULTIPLICATION

$x = ab$

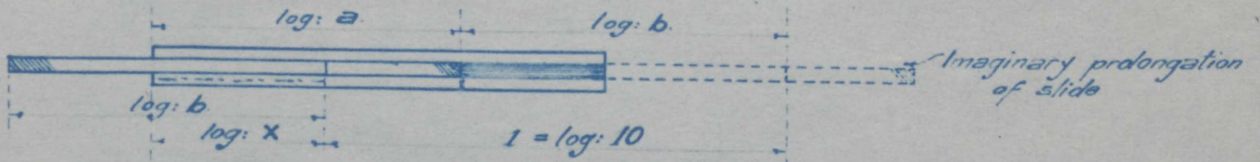


Using upper scales add 1 to caracteristic if result is read in second half.

25 x 0.3 = 7.5 Characteristic 1 + (-1) = 0

0.25 x 640 = 160, Result in second scale, Char: (-1) + 2 + 1 = 2

For the bottom scales add 1 to Characterist: if the result ought to be read beyond the scale, to get the figures in this case put right end of slide against multiplicand and read produce opposite multiplier on scale



For better understanding try left top scales; the right hand scales then show the prolongation of the rule.

(continued)

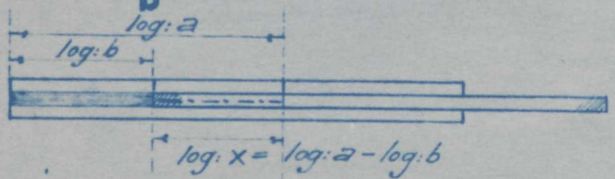
MULTIPLICATION (continued.)

$25 \times 0.3 = 7.5$ Result on scale, Character: $1 + (-1) = 0$

$0.25 \times 640 = 160$ Result beyond scale, put right end of slide to 25 on scale, against 64 on slide stands 16 on scale, Charact: $(-1) + 2 \underline{+1} = 2$. Or reverse order and put slide against 64 and read against 25 as before 16.

DIVISION (by Claudel's method)

$$x = \frac{a}{b}$$



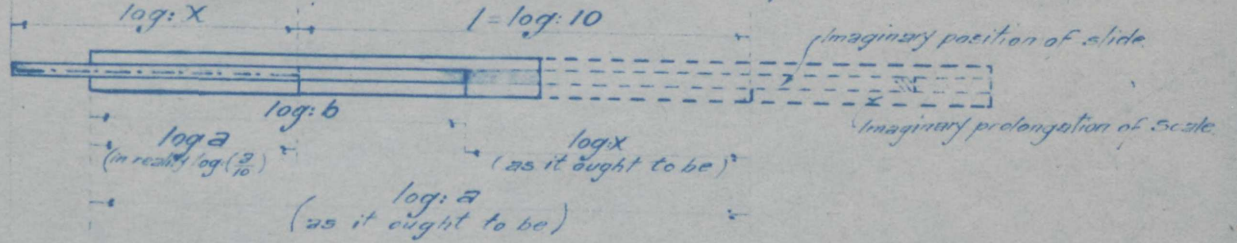
Top scales. Put left end of slide against divider and read figures of result on slide against dividend on scale. If the dividend is read in second part of scale deduct 1 from characteristic.

$\frac{640}{1.6} = 400$, Charact: $2 - 0 = 2$

(Continued)

$\frac{0.024}{800} = 0.00003$, Char: $(-2) - 2 - 1 = -5$

Bottom Scales, if the dividend ought to be read beyond the scale, put right end of slide against divider and read result on slide against dividend on scale; subtract 1 from characteristic in this case. A dividend 1 (in $\frac{1}{a}$) is to the right but in the imaginary prolongation of the scale, hence also here 1 must be subtracted from the Characteristic.

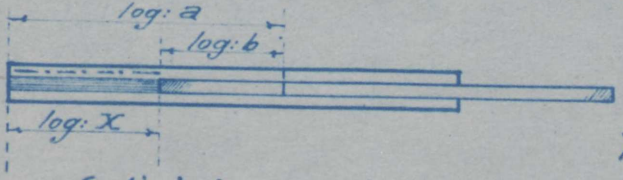


Above figure is better understood by using left top scales, the right hand scales then represent the imaginary prolongation.

$\frac{96}{0.016} = 6000$ Dividend on scale Charact: $1 - (-2) = 3$.

(Continued.)

$\frac{278.8}{0.082} = 3400$ Dividend beyond scale (as seen by putting left end of slide against 82). Hence put right end of slide against 82 on scale, and against 2788 on scale read 34 on slide. Characteristic $2-(-2)-\underline{1} = 3$.



This sketch shows another way of division, but to set divider on slide against dividend on scale is troublesome unless dividend first is marked by the sliding index. This method is of use in combined multiplications and divisions when the index anyhow is in continuous use and where we are willing to take the errors that arise by using it.

As the bottom scale has double the length it gives only half the error but a little more time

(continued)

DIVISION (continued)

is to be spent if the result is obtained by the imaginary prolongation and not seen in time.

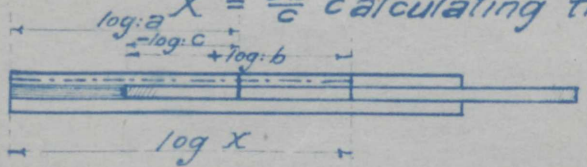
For rough work the top scales are preferable as well as for the aprenticeship.

NB. Det sidste er maaske ikke mere endelt, se Side 8 a-d
1-7-17

COMBINED MULTIPLICATION AND DIVISION OR CASES OF MORE THAN TWO FACTORS

may be executed by adding or subtracting one by one and marking each result by the index. If we divide every second time the result is obtained quicker as one move gives

$X = \frac{ab}{c}$ calculating thus $\frac{a}{c} \times b$, when using the bottom scale this can not always be done and using the top scales often only by using the second part. If in any case the result goes outside



(continued)

COMBINED MULTIPLICATION AND DIVISION OR CASES OF MORE THAN TWO FACTORS (continued)

the scale (bottom scale or top left half) add each time 1 to characteristic if running away to the right and subtract 1 from character: if overrunning to the left; if running outside the rule the opposite end of the slide has to be applied to the index.

It often accelerates the work to utilize the rule that the order of multiplications and divisions is arbitrary.

$\frac{3.2 \times 0.7}{0.016} = 140$ "TOP SCALES" Put 16 on slide against 32 on scale and read against 7 on slide answer 14 on scale, remembering that 14 is in second part and to the right Charact: $0 + (-1) - (-2) + 1 = 2$.

"BOTTOM SCALES" Put 16 on slide against 32 on scale, then there is no scale opposite 7 on slide, mark left end of slide by index and put right end of slide

(continued)

against index then opposite 7 on slide stands answer 14 on scale; having overrun to right we have to add 1 to the characteristic as above.

$\frac{36 \times 700 \times 40}{63 \times 0.08} = 200000$ "BOTTOM SCALE" Put 63 on slide against 36 on scale and mark 7 on slide by index ($\frac{36}{63} \times 7$ is done). Put 8 on slide against index and read against 4 on slide answer 200 on scale. Characteristic $1+2+1-1-(-2)=5$.

see page 8 a × b, 8 c × d.

SQUARES

This depends upon the upper scales being made each half the length of the bottom scales hence for same length ~~gives~~ double the reading or the square of any number of the bottom scale is read opposite in the upper scale against the end

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SQUARES (Continued.)

of the slide or the line on the index - remembering that when the reading is found in the second half 1 must be added to the characteristic. (The corresponding numbers in the two scales may be marked by end of slide or by the index.)

Exp: 1. $250^2 = 62500$. Against 25 on bottom scale is read 625 on 1st top scale, the characteristic of 250 is 2 hence of result $2 \times 2 = 4$.

Exp: 2. $0.045^2 = 0.00202$. Against 45 is read 202 on 2nd top scale, the characteristic of 0.045 is -2 hence $2 \times (-2) = -4$, the result is read in the 2nd top scale therefore add 1, or charact: of result $-4+1 = -3$

SQUARE ROOTS If the characteristic of the number is even use first and if odd use second top scale and read root opposite in bottom scale

(continued)

SQUARE ROOTS (continued)

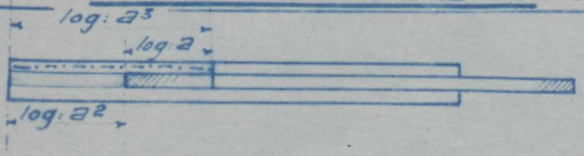
If 2nd top scale is used subtract 1 from character: before dividing by 2.

EXP: 1 $\sqrt{0.00095} = 0.0308$. Characteristic of number is -4 or even, read under 95 in 1st top scale 308 in bottom scale; characteristic of result $\frac{-4}{2} = -2$.

EXP: 2 $\sqrt{9500} = 97.5$. Characteristic of number is 3 or odd, read under 95 in 2nd scale 975. Characterist: of result $\frac{3-1}{2} = 1$

EXP: 3 $\sqrt{0.000095} = 0.00975$ Character: -5 odd and 975 is read as above. For result Character: $\frac{-5-1}{2} = -3$.

CUBES AND CUBE ROOTS



May be worked according to sketch remembering the 1 in the characteristic if 2nd top scale is used.

The higher numbers can-not be worked, already 5^3 would

(Continued.)

require a 3rd top scale.

Exp: $41.^3 = 69000$. Set end of slide against 41 on bottom scale and read against 41 on upper slide scale 69 on 2nd top scale. Characteristic: $3 \times 1 = 3$ plus 1 for second scale or $3+1=4$.

$\sqrt[3]{1728} = 12$ can be found by trial in moving slide until same reading, here 12, is obtained on bottom scale and upper slide scale. In general the slide rule is unpractical for cubes and cube roots.

See page 119 & 2.

SINES

The natural sine is obtained by moving the slide until the angle is opposite the corresponding line on the side of the slot on back of rule and then read the figures on the upper edge of the slide against the right hand end of the top scale. If read in the 1st scale of the slide put 0.0 before the figures and if in the

(Continued.)

SINES (continued)

2nd put 0. in front.

Exp:1 Sin: 20° = 0.34 Having set 20° against the mark in the slot, 34 is read in the second scale of the slide.

Exp: 2 Sin: 5° = 0.087 here the 87 is read in the 1st scale of the slide.

COSINES May be had by means of the sines because cos:α = Sin:$(90°-\alpha)$

TANGENTS UP TO 45° To find the natural tangent put angle against corresponding slot-mark on back of rule and read figures on top scale against right hand end of slide. If result is in 1st scale put 0.0 in front and in second scale put 0. in front of the figures.

Exp: Tang: 15° = 0.27 is read on the second scale.

For more than 45° must be used the trigonometrical detour tang:α = cotan:$(90°-\alpha) = \frac{1}{\tan(90-\alpha)}$.

Exp: Tang 85° = cotan:5° = $\frac{1}{\tan 5°} = \frac{1}{0.087} = 11.4$. Having set

(continued)

TANGENTS (continued)

5° against slot-mark is read 87 in 1st scale hence tang: 5° = 0.087. The $\frac{1}{0.087}$ is read on second slide scale against 1 at the commencement of the first top scale as 114 considering 1st top scale and 2nd slide scale as a slide rule for themselves, this gives the charact: to be 0-(-2) = 2 from which must be deducted 1 or 2-1=1; see description of division on top of page 4.

COTANGENTS See page 13a, 13b & 15 are found by cotan: α = tang: $(90° - \alpha)$

LOGARITHMS Put slide end against number read on bottom scale, turn rule over and read mantissa on back of slide against corresponding mark on slot.

Exp: log: 21 = 1.322 can be found thus.

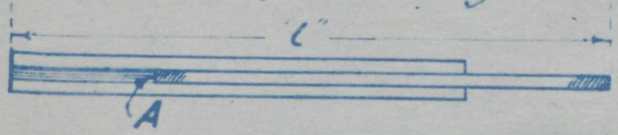
The logarithm found is the common Brigg's logar: of basis 10 See page 13b & 15

ORDINARY SCALES The machine divided c.m. scale serves for ordinary scaling on a drawing. The milled

(continued)

ORDINARY SCALES (continued)

scale on the back is for rougher measurement not exceeding length of rule, greater lengths are obtained by drawing out the slide and reading against its end in the bottom of the groove, thus length "l" is read at A.



THE UTILITY OF THE SLIDE RULE is great for multiplications and divisions that do not require more figures than the rule can give; in this case it is preferable to ordinary calculation with figures. It is handy as a check on ordinary calculations as it proves that the first figures and the decimal point are right.

For squares, square roots, trigonometric lines and logarithms it is usefull when no tables are at hand, else tables are generally preferable. However for square roots, as well as for cube roots if the rule has a triplicate scale, it may compete favourably with the tables 28-2-17 T.N.

Buenos Aires March 1905 Theodor Nielsen

FIGURES MARKED ON MANY "SLIDE RULES":

$\pi = 3.142$ $\frac{\pi}{4} = 0.785$ $\mathbb{C}\sqrt{\frac{4}{\pi}} = 1.128$ to find diameter $d = \mathbb{C}\sqrt{\text{Area}}$ for a circle. $\rho' = 3438'$ $\rho'' = 206265''$ giving respectively minutes or seconds in a circular arc of length equal to radius. Further for centesimal system 100° (not 90°) for right angle $\rho_{II} = 63.662$ new degrees

To find figures (number) corresponding to logarithms, set 301 on scale for log's at corresponding mark and find scale that gives the number 2, (which has the mantissa 301) Then this is the scale in which numbers must be read.

To find scales giving sine or tangent, remember that for one degree both are very near. $1\frac{3}{4}$ per cent, and for 10 degrees practically 10 times as much (you may read on the scale 174 and 176 in stead of the 175)
For small angles put 1° for sine or tangent (or scale for both) on mark and find $1\frac{3}{4}$ % = 0.0175 in a principal scale corresponding to end of other scale, then the scale thus found gives sine or tangent with decimal point corresponding to above figure (0.0 in front of reading) For larger angles put sine 10° or tangent 10° to mark and find 10 times $1\frac{3}{4}$ % or 0.175 proceed as above and fix decimal point corresponding to the 0.175 (0. in front of reading)

21-12-15
T. N.

Combined Multiplication and Division.

Using the bottom scales you will soon be able to formulate the following rules:

A. For each time the slide must be pushed to the left, when multiplying, add 1 to the characteristic.

B. For each time a multiplyer following a divider stands on the slide outside the rule, and for that reason the slide must be pushed to the right, subtract 1. Equally so if you finish with a division and read at right end of slide.

C. Resumen: for "special" moves of slide end to cursor, add 1 when you move to the left, and subtract one when you move slide to the right or read at its right hand end.

Example: $\frac{2 \times 3700 \times 0.145}{0.91} = 1180.$

Solution by alternate multiplication and division: Start with $\frac{2}{91}$ putting 91 on slide against 2 on the rule. The

next multiplier 37, on the slide, is outside the rule, therefore −1 to the characteristic. Move cursor to end of slide, pull slide to the right and set its commencement against the cursor. Then move cursor up to 37 on slide and this multiplication is finished. To multiply by 145 the slide must be pushed to the left, therefore +1 to characteristic resulting −1+1=0. With the end of the slide against cursor you read in front of 145 on the slide the resulting figures 118 on the rule. The characteristic is 0+3−1−(−1)=3 and the answer 1180.

Solution by using arbitrary order of multiplication and division: Start by 145×37 setting left end of slide against 145 on rule and then cursor against 37 on slide. Then divide by 91 putting 91 on slide against cursor, and finally multiply by 2 by reading in front of 2 the resulting figures 118 on rule. Answer 1180.—
The latter method gives less movement of slide and cursor and result at the distinct 2.

Note that, what we call the higher end of a scale, really is the lower end of the following, real or imaginary, scale. Neglecting this may cause a mistake of 1 in the characteristic.

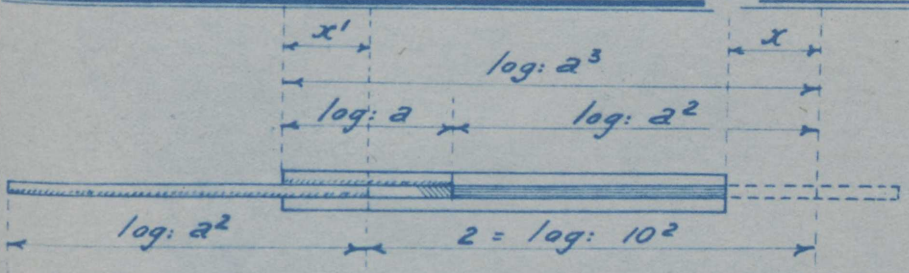
When using the upper double scales consider the left scales as the standard scales. Hence for a "special" move of the slide to the right, or when reading at the right end of the first scale, you get −1 to the characteristic, as for the bottom scales. A reading in the second scale gives +1 to the characteristic. A special move to the left of the slide gives +2 because the multiplier is in front of the imaginary prolongation of the rule

Ex: $\frac{0.62 \times 130 \times 94 \times 0.032 \times 8.4}{0.89 \times 110} = 20.8$

÷ er Divisions
Tegn fra Engelsk

Working thus $0.62 \div 0.89 \times 130 \div 110 \times 94 \times 0.032 \times 8.4$ the slide must be moved to the right for 13, hence −1. To multiply by 32, the slide must be pushed to the left,

CUBES AND CUBE ROOTS (Continued)



When higher numbers are to be cubed the slide can be reversed reading at x', instead of x outside the rule. It will be seen that the characteristic correction is 2.

Ex: $52.5^3 = 145000$. Set end of slide against 525 in the first of the upper scales of the rule; read, by sliding index in front of 525 on the bottom scale of the slide, the result 145 on the first of the upper scales of the rule. The characteristic of 52.5 is 1 hence here $3 \times 1 = 3$ to which must be added 2, as the slide was reversed. Hence characteristic of result $3 + 2 = 5$ giving the cube to be 145000.

NOTE: Above refers to the ordinarily used Mannheim

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Slide rule, a new type, the Rietz Slide-rule, has along the upper edge a triplicate division which can be worked together with the bottom scale by means of the sliding index thus allowing to work cubes and cube roots as easy as squares and square roots, it must only be remembered to adjust characteristic of cube by adding 1 if second and 2 if third of top scales are used.

Appendix to Page 13 before "cotangents"

Some recent german slide rules give the natural tangent by means of the bottom rule putting 0. in front of the reading. This is had on bottom of slide in front of left end of bottom rule, when the angle has been put against the mark on the back slot. For smaller angles than $5\frac{3}{4}^\circ$ ($0.1 = \tan 5^\circ 43'$) must be read sine in stead of tan., the difference being unnoticeable.

If more than one sine or tangent is wanted at a time reverse slide and read in the corresponding scale of the rule in front of the angle on the slide.

M.T.N.

TANGENTS (continued)

Some slide rules do not give sine and tangent for very small angles. For this case both are equal to the number of minutes in the angle times 0.000291 ($= \frac{1}{3438}$. The figure 3438 gives number of minutes and 206265 of seconds of the circular arc of length equal to radius; both are marked on many slide rules)

Ex: $631 \tan 0°30' = 631 \times 30 \times 0.000291 = 5.5$.

To multiply a figure with a sine or a tangent reverse the slide and put its commencement - or end - against the figure in the first of the upper scales and read result against the angle.

Ex: $46.5 \times \sin 40°30' = 30.2$. Put end of reversed slide against 46.5 in first top scale, and read against 40°30' the figure 302 in the same upper scale. As the sine is 0.64, found by the slide put in front of top scales of rule, the result will have the same number of figures in front of decimal point as 46.5

LOGARITHMS (continued)

The scale of logarithms can be used for finding roots or powers and for solving exponential equations. Ex: 1. $\sqrt[5]{0.00119} = 0.260$ To 119 the slide rule gives the mantissa 075, hence $\log \sqrt[5]{0.00119} = \frac{0.075-3}{5} = \frac{2.075-5}{5} = 0.415-1$. To the mantissa 415 the slide rule gives the number 260 and as the characteristic is −1, the result is 0.260

(continued)

LOGARITHMS (continued)

Ex: 2 $0.75^x = 0.316$, $\quad x \log 0.75 = \log 0.316$

$$x = \frac{\log 0.316}{\log 0.75} = \frac{0.499-1}{0.875-1} = \frac{-0.501}{-0.125} = \frac{0.501}{0.125} = 4.0$$